**Production Capacity Adjustment**

A manufacturing firm has discontinued the production of a certain unprofitable product line. This act created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table:

|  |  |
| --- | --- |
| ***Machine Type*** | ***Available Time***  *(In machine hours per week)* |
| Milling | 500 |
| Lathe | 350 |
| Grinder | 150 |

The number of machine hours required for each unit of the respective products is

**Productivity coefficient (in machine hours per unit)**

|  |  |  |  |
| --- | --- | --- | --- |
| ***Machine Type*** | ***Product 1*** | ***Product 2*** | ***Product 3*** |
| Milling | 9 | 3 | 5 |
| Lathe | 5 | 4 | 0 |
| Grinder | 3 | 0 | 2 |

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week. The unit profit would be $30, $12, and $15, respectively, on products 1, 2, and 3. The objective is to determine how much of each product the firm should produce to maximize profit.

We will now form the LP model for this problem

In all LPs, it is important to identify the decision variables first. In this problem, the variables management can control directly are the number of products 1, 2, and 3. The decision variables are labeled as follows:

*X*1 = units of product 1 to produce

X2 = units of product 2 to produce

X3 = units of product 3 to produce

Management needs to find the best combinations of products 1, 2, and 3 to produce in order to maximize profit. Profit is determined as follows:

30X1 + 12X2 + 15X3 = Profit

Production is limited by the following:

**Milling:** 9X1 + 3X2 + 5X3 ≤ 500

**Lathe:** 5X1 + 4X2  ≤ 350

**Grinder:** 9X1 + 5X3 ≤ 150

In addition the problem states sales production for product 3 is limited to 20 units per week:

X3 ≤ 20

Lastly, there cannot be production of negative amounts of product 1, 2, or 3.

X1 ≥ 0, X2 ≥ 0, X3 ≥ 0

Using Excel’s built in solver yields the following results:

X1 = 26.19, X2 = 54.76, X3 = 20

Profit = 1742.857

Having calculated the solution, we would now like to perform a sensitivity analysis of the solution, which is how changes in the coefficients and constraints of the problem affect the optimal basis we have found. Since the numbers defining LP model is subject to a variety of errors in the real world, sensitivity analysis is a crucial part of solving LP models.



**Reduced Cost and Range**

The reduced cost of a product refers to the increase (for maximization problems) or decrease (for minimization problems) of an objective value coefficient in order for the corresponding variable to be in the optimal solution. With the current model, the optimal basis includes all 3 different products. Therefore the reduced cost for each of the 3 products is zero. However if the unit profit of any of the products change, the optimal basis potential could change as well. The original coefficients of the objective function is 30, 12 and 15 which corresponds with product 1, 2, 3 respectively. For each coefficient there is a range where the solution basis remains optimal. For example, if the unit profit for product 1 remains between 3.75 (30+.75) and 15 (30-15), then the original solution basis remains optimal, provided all other variables do no change. It is important to note that if more than unit profits change simultaneously, the optimum basis could change, even if both changes are within their respective ranges. This same idea applies for products 2 and 3.

**Shadow Price**

The shadow price of a resource refers to the change in the profit per unit increase of the restraint of that resource, all else held equal. So in our example, grinder has a shadow price of 0. This means that even if we were to increase the allowable grinder time to, we would see no increase in profit. If we were to devote more time to a particular resource, it may be a good idea for management to devote more available time to the item with the highest shadow price. In this case, milling has the highest shadow price. If we increase the allowable milling time by 1 hour, we would see an increase in profit of 2.857 dollars.

The corresponding dual of the LP is:

Minimize 500Y1 + 350Y2 + 150Y3 + 20Y4 = Z

9Y1 + 5Y2 + 3Y3  ≥ 30

3Y1 + 4Y2  ≥ 12

5Y1 + 2Y3 + Y4 ≥ 15

Y1 ≥ 0, Y2 ≥ 0, Y3 ≥ 0, Y4 ≥ 0

The solution is:

Y1 = 4.8, Y2 ≥ 3.378, Y3 ≥ 1.43, Y4 ≥ 0.19

Z = 3800.875

We now have the optimum solution to the dual which is an upper bound to the solution of the primal. We can gain a profit of $3800.875 at most.